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Ferromagnetic fixed point of the Kondo model in a Luttinger liquid

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Abstract

The Kondo effect in a Luttinger liquid is studied using the renormalization group method. By renormalizing the boson fields, scaling equations to the second order for an arbitrary Luttinger interaction are obtained. For the ferromagnetic Kondo coupling, a spin bound state (triplet) can be realized without invoking a nearest neighbour spin interaction in agreement with the recent Bethe *ansatz* calculation. The scaling theory in the presence of the scalar potential shows that there is no interplay between the magnetic and non-magnetic interaction. Also a study on the crossover behaviour of the Kondo temperature between the exponential and the power-law type is presented.

1. Introduction

The Kondo problem in a one-dimensional quantum system has attracted substantial interest in connection with the recent rapid development of the nanofabrication technology. In one dimension, the interacting electron system is described by the Tomonaga–Luttinger liquid theory [1–5], whose low-energy excitations are not quasiparticles but collective charge and spin density fluctuations. The magnetic impurity effect in this non-Fermi liquid was first studied by Lee and Toner using a scaling analysis on the kink gas action [6]. They obtained the scaling equation to the first order of coupling constant in the weak coupling regime. Furusaki and Nagaosa (FN) extended the study and obtained a set of scaling equations up to the second order using the poor man's scaling theory [7]. In their study, FN proposed an interesting conjecture that even a ferromagnetic Kondo impurity as well as an antiferromagnetic one would be completely screened. In the antiferromagnetic coupling case, the coupling constant flows to a strong coupling regime and, thus, the magnetic impurity and a conduction electron form a singlet. But, in the ferromagnetic case, it is not clear whether it flows into a strong coupling regime or not. In order to clarify the situation, FN considered coupling of impurity spins not only with the same site electrons but also with the nearest neighbour conduction electrons. In this picture, the impurity and three conduction electrons form a singlet composite. However, a recent Bethe *ansatz* (BA) result by Wang and Voit showed that spins form a triplet for the ferromagnetic coupling case [8]. These conflicting results call for a more detailed renormalization group (RG) analysis to clarify the physical situation.

In this paper, we carry out an RG analysis which goes beyond the poor man's scaling scheme in the presence of the Kondo interaction. We bosonize the Kondo interaction term using the Abelian bosonization. Carrying out a full RG calculation for the second-order cumulant, we obtain a set of general scaling equations valid for an arbitrary strength of the Luttinger interaction. We show that the present RG calculation confirms the calculation drawn from the BA calculation using an open boundary condition.

Another recent BA study of the same problem by Wang and Eckern showed that there is a competition between the Kondo coupling and the impurity potential [9]. When the impurity potential is dominant, the system is shown to flow to a weak coupling fixed point. However, when the magnetic interaction is dominant, a spin complex is shown to be formed. In order to investigate this problem, we bosonize the potential scattering term and perform a scaling calculation. The result shows that the magnetic interaction and the potential scattering do not interplay in the scaling procedure and flow independently.

In the first-order scaling, the Kondo temperature in the Luttinger can be calculated analytically to show a power-law behaviour in contrast to the exponential one of the conventional Kondo model [6]. However, it is not possible to show the crossover from the Fermi to the Luttinger liquid in the first order. In this paper, we study the crossover behaviour of the Kondo temperature from the exponential to the power-law type as a function of the Luttinger interaction, by solving the RG equations.

2. Bosonization of the Kondo Hamiltonian and the partition function

Introducing the two phase fields,

$$\phi_\nu = -\frac{\mathrm{i}\pi}{L}\sum_{p\neq 0}\frac{\mathrm{e}^{-\alpha|p|/2-\mathrm{i}px}}{p}(\nu_+(p)+\nu_-(p)) - (N_{+\nu}+N_{-\nu})\frac{\pi x}{L}$$
$$\theta_\nu = \frac{\mathrm{i}\pi}{L}\sum_{p\neq 0}\frac{\mathrm{e}^{-\alpha|p|/2-\mathrm{i}px}}{p}(\nu_+(p)-\nu_-(p)) + (N_{+\nu}-N_{-\nu})\frac{\pi x}{L} \tag{1}$$

the one-dimensional model Hamiltonian with the forward electron–electron scattering is written by

$$H = \frac{1}{2\pi}\sum_\nu\int \mathrm{d}x\left(v_\nu\eta_\nu\pi^2\pi_\nu^2(x) + \frac{v_\nu}{\eta_\nu}\left(\frac{\partial\phi_\nu(x)}{\partial x}\right)^2\right) \tag{2}$$

where $\nu = \rho$ or σ, + (−) means the right (left) going mode and the parameters v_ν and η_ν are given by

$$v_\nu = \sqrt{\left(v_F + \frac{g_{4\nu}}{\pi}\right)^2 - \left(\frac{g_{2\nu}}{\pi}\right)^2} \qquad \eta_\nu = \sqrt{\frac{v_F + ((g_{4\nu}-g_{2\nu})/\pi)}{v_F + ((g_{4\nu}+g_{2\nu})/\pi)}}. \tag{3}$$

Here, g_2 and g_4 are the dimensionless coupling parameters [3]. The fields ϕ_ν and π_ν satisfy the canonical boson commutation relation

$$[\phi_\nu(x), \pi_{\nu'}(x')] = \mathrm{i}\delta_{\nu\nu'}\delta(x-x')$$
$$[\phi_\nu(x), \theta_{\nu'}(x')] = \mathrm{i}\frac{\pi}{2}\delta_{\nu\nu'}\,\mathrm{sign}(x'-x) \qquad \theta_\nu(x) = \pi\int_{-\infty}^{x}\pi_\nu(y)\,\mathrm{d}y. \tag{4}$$

Alternatively, this Hamiltonian can also be obtained directly from the fermion representation of the one dimensional interacting electron system using

$$\psi_{rs}(x) = \lim_{\alpha\to 0} \frac{e^{ir(k_F-\frac{\pi}{L})x}}{\sqrt{2\pi\alpha}} \eta_{rs} \exp\left[-\frac{i}{\sqrt{2}}(r\phi_\rho(x) - \theta_\rho(x) + s(r\phi_\sigma(x) - \theta_\sigma(x)))\right] \quad (5)$$

where η_{rs} is the Majorana fermion operator which satisfies the following relations [5]:

$$[\eta_{rs}, \eta_{r's'}] = 2\delta_{rr'}\delta_{ss'}$$
$$\eta_{+\downarrow}\eta_{+\uparrow} = \eta_{-\downarrow}\eta_{-\uparrow} \qquad \eta_{+\downarrow}\eta_{-\uparrow} = -\eta_{-\downarrow}\eta_{+\uparrow} \qquad \eta_{+\uparrow}\eta_{-\uparrow} = -\eta_{+\downarrow}\eta_{-\downarrow}\cdots. \quad (6)$$

If a single magnetic impurity is introduced in one-dimensional interacting electron system, the Kondo interaction term is given by

$$\begin{aligned} H_K &= J\vec{S}\vec{s}(0) \\ &= J_z S_z s_z(0) + \tfrac{1}{2} J_\perp (S_+ s_-(0) + S_- s_+(0)) \end{aligned} \quad (7)$$

where $\vec{s} = \frac{1}{2}\sum_{rr'\sigma\sigma'} \psi^\dagger_{r\sigma}\vec{\sigma}_{\sigma\sigma'}\psi_{r'\sigma'}$ and $s_\pm = s_x \pm i s_y$. Using the relations of the Majorana fermions, equation (6), and the bosonization formula of the fermion operators, we obtain the bosonized Kondo Hamiltonian [10],

$$\begin{aligned} H_K = J\vec{S}\vec{s} &= \frac{S_z}{2\pi\alpha}(J_{zF}\alpha\partial_x\sqrt{2}\phi_\sigma(0) + 2iJ_{zB}\eta_{+\uparrow}\eta_{-\uparrow}\sin(\sqrt{2}\phi_\rho(0))\cos(\sqrt{2}\phi_\sigma(0))) \\ &+ \frac{S_+}{2\pi\alpha}\,e^{\sqrt{2}i\theta_\sigma(0)}(J_{\perp F}\eta_{+\downarrow}\eta_{+\uparrow}\cos(\sqrt{2}\phi_\sigma(0)) + iJ_{\perp B}\eta_{+\downarrow}\eta_{-\uparrow}\sin(\sqrt{2}\phi_\rho(0))) \\ &+ \frac{S_-}{2\pi\alpha}\,e^{-\sqrt{2}i\theta_\sigma(0)}(J_{\perp F}\eta_{+\uparrow}\eta_{+\downarrow}\cos(\sqrt{2}\phi_\sigma(0)) - iJ_{\perp B}\eta_{-\uparrow}\eta_{+\downarrow}\sin(\sqrt{2}\phi_\rho(0))). \end{aligned} \quad (8)$$

The partition function of the system at temperature $T = 1/\beta$ is

$$\begin{aligned} Z &= \int D\phi_\rho D\phi_\sigma D\theta_\rho D\theta_\sigma\, e^{-S} \\ S &= \int dx \int d\tau\,(L_0 + L_K) \\ L_0 &= \sum_{\nu=\rho,\sigma}\left(i\partial_\tau\phi_\nu(x,\tau)\pi_\nu(x,\tau) + \frac{1}{2\pi}\left(v_\nu\eta_\nu\pi^2\pi_\nu^2(x,\tau) + \frac{v_\nu}{\eta_\nu}\left(\frac{\partial\phi_\nu(x,\tau)}{\partial x}\right)^2\right)\right) \\ L_K &= H_K(\phi_\nu(0,\tau), \theta_\nu(0,\tau)) \end{aligned} \quad (9)$$

where the integration is over the bosonic fields $\phi_\nu(x,\tau)$ and $\theta_\nu(x,\tau)$ with imaginary time τ running from 0 to β.

3. Renormalization analysis

First, we divide the phase fields ϕ_ν into slow and fast mode:

$$\begin{aligned} \phi_\nu(\tau) &= \phi_{\nu s}(\tau) + \phi_{\nu f}(\tau) \\ \phi_{\nu s}(\tau) &= \frac{1}{\beta}\sum_{|\omega_n|<\mu}\tilde{\phi}_\nu(\omega)\,e^{-i\omega\tau} \\ \phi_{\nu f}(\tau) &= \frac{1}{\beta}\sum_{\mu<|\omega_n|<\lambda}\tilde{\phi}_\nu(\omega)\,e^{-i\omega\tau}. \end{aligned} \quad (10)$$

The average over the fast mode of the partition function is carried out, using the cumulant expansion,

$$\begin{aligned} Z &= Z_0\langle e^{-S_K}\rangle_0 \\ &= Z_0\int D\phi_{\nu s}D\theta_{\nu s}\exp[-S_0(\phi_{\nu s},\theta_{\nu s})]\exp\left[-\langle S_K\rangle_0^f + \frac{1}{2}(\langle {S_K}^2\rangle_0^f - \langle S_K\rangle_0^{f\,2}) + \cdots\right] \end{aligned} \quad (11)$$

where $Z_0 = \int D\phi_\nu D\theta_\nu \, e^{-S_0}$, f indicates an average over the fast mode which will be omitted hereafter and $\langle \cdots \rangle_0$ represents an average over Z_0.

First, we consider the first-order forward longitudinal scattering term

$$\left\langle \int d\tau \; J_{zF} \alpha \partial_x \sqrt{2} \phi_\sigma(\tau) \right\rangle = \int d\tau \; J_{zF} \alpha (\partial_x \sqrt{2} \phi_{\sigma s}(\tau) + \langle \partial_x \sqrt{2} \phi_{\sigma f}(\tau) \rangle). \quad (12)$$

The second term in the right side vanishes because it is an average of an odd function. We, thus, conclude that $\delta J_{zF} = 0$ in the first order. The backward longitudinal scattering part is scaled as follows:

$$\begin{aligned} &\int d\tau \; J_{zB} \eta_{+\uparrow} \eta_{-\uparrow} \langle \sin(\sqrt{2}\phi_\rho(\tau)) \rangle \langle \cos(\sqrt{2}\phi_\sigma(\tau)) \rangle \\ &\quad = \left(\frac{\mu}{\lambda}\right)^{(\eta_\rho/2)+(\eta_\sigma/2)} \int d\tau \; J_{zB} \eta_{+\uparrow} \eta_{-\uparrow} \sin(\sqrt{2}\phi_{\rho s}(\tau) \cos(\sqrt{2}\phi_{\sigma s}(\tau)). \end{aligned} \quad (13)$$

In the above, we utilized the fact that the charge and spin degrees are separated. The rescaling procedure,

$$J_{zB}(\mu) = \left(\frac{\mu}{\lambda}\right)^{((\eta_\rho+\eta_\sigma)/2)-1} J_{zB}(\lambda) \quad (14)$$

gives

$$\frac{\delta J_{zB}}{J_{zB}} = \left(\frac{\eta_\rho + \eta_\sigma}{2} - 1\right) \frac{\delta\lambda}{\lambda} \quad (15)$$

where $\mu = \lambda + \delta\lambda$ and $\delta l = -\delta\lambda/\lambda = -\delta \ln \lambda$. Thus, we have in the first order

$$\begin{aligned} \frac{dJ_{zF}}{dl} &= 0 \\ \frac{dJ_{zB}}{dl} &= \left(1 - \frac{\eta_\rho + \eta_\sigma}{2}\right) J_{zB}. \end{aligned} \quad (16)$$

The scaling equations for the other scattering terms can be similarly obtained:

$$\begin{aligned} \frac{dJ_{\perp F}}{dl} &= \left[1 - \left(\frac{1}{2\eta_\sigma} + \frac{\eta_\sigma}{2}\right)\right] J_{\perp F} \\ \frac{dJ_{\perp B}}{dl} &= \left[1 - \left(\frac{1}{2\eta_\sigma} + \frac{\eta_\rho}{2}\right)\right] J_{\perp B}. \end{aligned} \quad (17)$$

These equations are in agreement with those of Lee and Toner [6].

The second-order cumulant is given by $-\frac{1}{2}(\langle S_K^2 \rangle - \langle S_K \rangle^2)$, where the $\langle S_K \rangle^2$ term is to eliminate unconnected diagrams. We consider one of the $J_{zB} J_{\perp F}$ terms which is given by

$$\begin{aligned} &\int d\tau \int d\tau' \frac{S_z S_+}{(2\pi\alpha)^2} J_{zB} 2i \eta_{+\uparrow} \eta_{-\uparrow} J_{\perp F} \eta_{+\downarrow} \eta_{+\uparrow} \\ &\quad \times (\langle \sin(\sqrt{2}\phi_\rho(\tau)) \cos(\sqrt{2}\phi_\sigma(\tau)) \, e^{\sqrt{2} i \theta_\sigma(\tau')} \cos(\sqrt{2}\phi_\sigma(\tau')) \rangle \\ &\quad - \langle \sin(\sqrt{2}\phi_\rho(\tau)) \cos(\sqrt{2}\phi_\sigma(\tau)) \rangle \langle e^{\sqrt{2} i \theta_\sigma(\tau')} \cos(\sqrt{2}\phi_\sigma(\tau')) \rangle). \end{aligned} \quad (18)$$

In order to evaluate this expression, we need the two point correlation function [11–14]

$$\begin{aligned} G(x, \tau) &= \langle \phi(x, \tau) \phi(0, 0) \rangle \\ &= \int \frac{dq}{2\pi} \int \frac{d\omega}{2\pi} \, e^{-iqx} \, e^{i\omega\tau} \pi / [((1/v\eta)\omega^2) + ((v/\eta)q^2)] \\ G(\tau) &\equiv G(0, \tau) \\ &= \begin{cases} \dfrac{\eta}{2} K_0(\mu\tau) & \text{for } \lambda\tau \gg 1 \\ \dfrac{\eta}{2} \ln \dfrac{\lambda}{\mu} & \text{for } \lambda\tau \ll 1 \end{cases} \end{aligned} \quad (19)$$

where K_0 is the modified Bessel function of the second kind. $G(\tau)$ decays exponentially with the renormalized lattice spacing $1/\mu$, and decreases logarithmically for small $\lambda\tau$. Thus, we regard $G(\tau)$ as short ranged and, thus, expand the cosine terms around zero. It is known that the higher harmonic $\cos(2\sqrt{2}\phi(0))$ is irrelevant and the $((\partial\phi(\tau)/\partial\tau)|_{\tau=0})^2$ term, which is the most relevant term in the expansion can, also be shown to be irrelevant by the power counting [11]. Short time cut-off, $\tau_0 \sim \alpha/v_F$, merely introduces an overall constant, which does not affect the flow of the parameter. Therefore equation (18) is reduced to

$$\int \mathrm{d}\tau \frac{\alpha}{v_F}\frac{1}{(2\pi\alpha)^2}\frac{1}{2}S_+ J_{zB}J_{\perp F}(-\mathrm{i}\eta_{+\downarrow}\eta_{-\uparrow})\left(\frac{\mu}{\lambda}\right)^{(\eta_\rho/2)+(1/2\eta_\sigma)+\eta_\sigma}\left(\left(\frac{\mu}{\lambda}\right)^{-\eta_\sigma}-1\right) \times \mathrm{e}^{\sqrt{2}\mathrm{i}\theta_\sigma(\tau)}\sin(\sqrt{2}\phi_\rho(\tau)). \tag{20}$$

Collecting other terms of the second-order cumulant and rescaling as before, we have for the transverse backward part,

$$\int \mathrm{d}\tau \frac{S_+}{2\pi\alpha}\mathrm{i}\eta_{+\downarrow}\eta_{-\uparrow}\eta_\sigma J_{zB}J_{\perp F}\frac{\mathrm{d}\lambda}{\lambda}\,\mathrm{e}^{\sqrt{2}\mathrm{i}\theta_{\sigma s}(\tau)}\sin(\sqrt{2}\phi_{\rho s}(\tau)) \tag{21}$$

which renormalizes the coupling constant $J_{\perp B}$ of the Kondo term. Similarly, we have for the transverse forward scattering part, which renormalizes the coupling constant $J_{\perp F}$,

$$\int \mathrm{d}\tau \frac{S_+}{2\pi\alpha}\eta_{+\downarrow}\eta_{+\uparrow}\eta_\rho J_{zB}J_{\perp B}\frac{\mathrm{d}\lambda}{\lambda}\,\mathrm{e}^{\sqrt{2}\mathrm{i}\theta_\sigma(\tau)}\cos(\sqrt{2}\phi_\sigma(\tau)). \tag{22}$$

The same scaling process on the S_- terms gives the same renormalization for both $J_{\perp B}$ and $J_{\perp F}$.

However, the scaling process involving the descendant field terms, $J_{zF}\alpha\partial_x\sqrt{2}\phi_\sigma(\tau)$, is somewhat different. One example of such a term is the second-order cumulant for the transverse forward scattering, which is given by

$$\begin{aligned}
&\frac{S_z S_+}{(2\pi\alpha)^2}J_{zF}J_{\perp F}\eta_{+\downarrow}\eta_{+\uparrow}(\langle\alpha\partial_x\sqrt{2}\phi_\sigma(\tau)\,\mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(\tau')}\cos(\sqrt{2}\phi_\sigma(\tau'))\rangle \\
&\qquad -\langle\alpha\partial_x\sqrt{2}\phi_\sigma(\tau)\rangle\langle\mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(\tau')}\cos(\sqrt{2}\phi_\sigma(\tau'))\rangle) \\
&\qquad = \frac{S_z S_+}{(2\pi\alpha)^2}J_{zF}J_{\perp F}\eta_{+\downarrow}\eta_{+\uparrow}\langle\alpha\partial_x\sqrt{2}\phi_{\sigma f}(\tau)\,\mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(\tau')}\cos(\sqrt{2}\phi_\sigma(\tau'))\rangle.
\end{aligned} \tag{23}$$

Here, we note that

$$\langle\alpha\partial_x\sqrt{2}\phi_{\sigma f}(\tau)\,\mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(\tau')}\,\mathrm{e}^{\mathrm{i}\sqrt{2}\phi_\sigma(\tau')}\rangle = \lim_{\epsilon\to 0}\frac{1}{\mathrm{i}\epsilon}\frac{\partial}{\partial x}\langle\alpha\,\mathrm{e}^{\mathrm{i}\epsilon\sqrt{2}\phi_{\sigma f}(x,\tau)}\,\mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(0,\tau')}\,\mathrm{e}^{\mathrm{i}\sqrt{2}\phi_\sigma(0,\tau')}\rangle|_{x=0}. \tag{24}$$

Using $\mathrm{e}^{A+B} = \mathrm{e}^A\,\mathrm{e}^B\,\mathrm{e}^{-[A,B]/2}$, the above expression takes the form

$$\frac{\alpha}{\mathrm{i}}(2\partial_x G_{\phi_{\sigma f}}(x,\tau) - 2\partial_x\langle\phi_{\sigma f}(x,\tau)\theta_{\sigma f}(0,0)\rangle)|_{x=0}\,\mathrm{e}^{-G_{\theta_{\sigma f}}(0,0)-G_{\theta_{\phi f}}(0,0)}. \tag{25}$$

In order to calculate $\partial_x\langle\phi_{\sigma f}(x,\tau)\theta_{\sigma f}(0,0)\rangle$, we use the relations between density field, ϕ_σ, and current field, θ_σ [3, 14],

$$\begin{aligned}
-\frac{\mathrm{i}}{\eta_\sigma}\frac{\partial\phi_\sigma}{\partial(v_\sigma\tau)} &= \frac{\partial\theta_\sigma}{\partial x} \\
-\mathrm{i}\eta_\sigma\frac{\partial\theta_\sigma}{\partial(v_\sigma\tau)} &= \frac{\partial\phi_\sigma}{\partial x}.
\end{aligned} \tag{26}$$

Then, we have

$$\begin{aligned}
\alpha\partial_x\langle\phi_{\sigma f}(x,\tau)\theta_{\sigma f}(0,0)\rangle &= \alpha\eta_\sigma\frac{\partial}{\partial(v_\sigma\tau)}\langle\theta_{\sigma f}(x,\tau)\theta_{\sigma f}(0,0)\rangle \\
&= \frac{1}{2}\frac{d\lambda}{\lambda}
\end{aligned} \tag{27}$$

using equation (19) with $1/\eta \to \eta$ and the short-ranged nature of the correlation function $G(\tau)$. Substituting this result into equation (23), we obtain

$$\frac{1}{2\pi\alpha} S_+ \eta_{+\downarrow}\eta_{+\uparrow} \left(\frac{1}{2\pi v_F} J_{zF} J_{\perp F} \frac{\mathrm{d}\lambda}{\lambda}\right) \mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(\tau)} \cos(\sqrt{2}\phi_\sigma(\tau)) + \frac{1}{2\pi\alpha} S_+ \eta_{+\downarrow}\eta_{-\uparrow} \left(\frac{1}{2\pi v_F} J_{zF} J_{\perp B} \frac{\mathrm{d}\lambda}{\lambda}\right) \mathrm{e}^{\mathrm{i}\sqrt{2}\theta_\sigma(\tau)} \sin(\sqrt{2}\phi_\rho(\tau)) \tag{28}$$

for the renormalization of $J_{\perp F}$ and $J_{\perp B}$.

The longitudinal scattering parameter is scaled by consecutive transverse scattering processes. The forward scattering part which contains $J_{\perp F}^2$ is

$$\frac{S_+S_-}{(2\pi\alpha)^2} J_{\perp F}^2 \eta_{+\downarrow}\eta_{+\uparrow}\eta_{+\uparrow}\eta_{+\downarrow} [\langle \mathrm{e}^{\sqrt{2}\mathrm{i}\theta_\sigma(\tau)} \cos(\sqrt{2}\phi_\sigma(\tau))\, \mathrm{e}^{-\sqrt{2}\mathrm{i}\theta_\sigma(\tau')} \cos(\sqrt{2}\phi_\sigma(\tau'))\rangle - \langle \mathrm{e}^{\sqrt{2}\mathrm{i}\theta_\sigma(\tau)} \cos(\sqrt{2}\phi_\sigma(\tau))\rangle \langle \mathrm{e}^{-\sqrt{2}\mathrm{i}\theta_\sigma(\tau')} \cos(\sqrt{2}\phi_\sigma(\tau'))\rangle]. \tag{29}$$

The typical relevant term is given by

$$\mathrm{e}^{\sqrt{2}\mathrm{i}\theta_\sigma(\tau)}\, \mathrm{e}^{-\sqrt{2}\mathrm{i}\theta_\sigma(\tau')}\, \mathrm{e}^{-\sqrt{2}\mathrm{i}\phi_\sigma(\tau)}\, \mathrm{e}^{\sqrt{2}\mathrm{i}\phi_\sigma(\tau')}. \tag{30}$$

Other terms containing higher harmonics, $\mathrm{e}^{2\sqrt{2}\mathrm{i}\theta_\sigma(\tau)}$ or $\mathrm{e}^{2\sqrt{2}\mathrm{i}\phi_\sigma(\tau)}$ are irrelevant [11, 15]. Separating the fields into the fast and slow mode, we average on the fast mode to obtain a short range correlation and expand the slow mode to obtain

$$1 + \mathrm{i}\sqrt{2}\partial_\tau \theta_{\sigma s}(\tau)|_{\tau=0}\tau + \mathrm{i}\sqrt{2}\partial_\tau \phi_{\sigma s}(\tau)|_{\tau=0}\tau + \cdots. \tag{31}$$

We note that $\partial_\tau\theta_{\sigma s}(\tau)$ gives the longitudinal scattering contribution, $\partial_x\phi_{\sigma s}$, through equation (26), while $\partial_\tau\phi_{\sigma s}$ term is cancelled by other terms. Including the contributions from the S_-S_+ term, we obtain

$$\int \mathrm{d}\tau \left[\frac{1}{2\pi\alpha}\frac{S_z}{2\pi v_F}\frac{1}{\eta_\sigma}\left(\frac{1}{2\eta_\sigma} + \frac{\eta_\sigma}{2}\right) J_{\perp F}^2 \frac{\mathrm{d}\lambda}{\lambda} \alpha\partial_x\sqrt{2}\phi_{\sigma s}(\tau) + \frac{1}{2\pi\alpha}\frac{S_z}{2\pi v_F}\frac{1}{\eta_\sigma}\left(\frac{1}{2\eta_\sigma} + \frac{\eta_\rho}{2}\right) J_{\perp B}^2 \frac{\mathrm{d}\lambda}{\lambda} \alpha\partial\sqrt{2}\phi_{\sigma s}(\tau)\right] \tag{32}$$

for the $J_{\perp B}^2$ contribution to the J_{zF} renormalization. Consecutive transverse scattering also renormalizes the longitudinal backward term, J_{zB}, similarly;

$$\int \mathrm{d}\tau\, \frac{S_z}{2\pi\alpha} 2\mathrm{i}\eta_{+\uparrow}\eta_{-\uparrow} \frac{2}{2\pi v_F}\eta_\sigma J_{\perp F} J_{\perp B} \frac{\mathrm{d}\lambda}{\lambda} \sin(\sqrt{2}\phi_\rho(\tau)) \cos(\sqrt{2}\phi_\sigma(\tau)). \tag{33}$$

The last term to be considered in the second-order cumulant expansion is the $S_z^2 J_{zF}^2$ term. Scaling of this term, however, does not contribute to the magnetic interaction and, also, does not produce any relevant term for non-magnetic interaction. Collecting the results together, we now have the scaling equations to the second order:

$$\begin{aligned}
\frac{\mathrm{d}J_{zF}}{\mathrm{d}l} &= \frac{1}{2\pi v_F}\left(\frac{1}{2\eta_\sigma}\left(\frac{1}{\eta_\sigma} + \eta_\sigma\right) J_{\perp F}^2 + \frac{1}{2\eta_\sigma}\left(\frac{1}{\eta_\sigma} + \eta_\rho\right) J_{\perp B}^2\right) \\
\frac{\mathrm{d}J_{zB}}{\mathrm{d}l} &= \left(1 - \frac{\eta_\sigma - \eta_\rho}{2}\right) J_{zB} + \frac{1}{2\pi v_F} 2\eta_\sigma J_{\perp F} J_{\perp B} \\
\frac{\mathrm{d}J_{\perp F}}{\mathrm{d}l} &= \left(1 - \frac{\eta_\sigma + (1/\eta_\sigma)}{2}\right) J_{\perp F} + \frac{1}{2\pi v_F}(J_{zF} J_{\perp F} + \eta_\rho J_{zB} J_{\perp B}) \\
\frac{\mathrm{d}J_{\perp B}}{\mathrm{d}l} &= \left(1 - \frac{\eta_\sigma - \eta_\rho}{2}\right) J_{\perp B} + \frac{1}{2\pi v_F}(J_{zF} J_{\perp B} + \eta_\sigma J_{zB} J_{\perp F}).
\end{aligned} \tag{34}$$

These equations can be simplified assuming the SU(2) symmetry for the conduction electrons ($\eta_\sigma = 1$),

$$\frac{\mathrm{d}(\rho_0 J_{zF})}{\mathrm{d}l} = (\rho_0 J_{\perp F})^2 + \left(\frac{1+\eta_\rho}{2}\right)(\rho_0 J_{\perp B})^2$$
$$\frac{\mathrm{d}(\rho_0 J_{zB})}{\mathrm{d}l} = \left(\frac{1-\eta_\rho}{2}\right)(\rho_0 J_{zB}) + 2(\rho_0 J_{\perp F})(\rho_0 J_{\perp B})$$
$$\frac{\mathrm{d}(\rho_0 J_{\perp F})}{\mathrm{d}l} = (\rho_0 J_{zF})(\rho_0 J_{\perp F}) + \eta_\rho(\rho_0 J_{zB})(\rho_0 J_{\perp B})$$
$$\frac{\mathrm{d}(\rho_0 J_{\perp B})}{\mathrm{d}l} = \left(\frac{1-\eta_\rho}{2}\right)(\rho_0 J_{\perp B}) + (\rho_0 J_{zF})(\rho_0 J_{\perp B}) + (\rho_0 J_{zB})(\rho_0 J_{\perp F}) \quad (35)$$

where ρ_0 ($=1/2\pi v_F$) is proportional to the density of state. As we see later, for ferromagnetic J, the triplet ground state can be formed due to the interaction parameter η_ρ in the second order term. In 1D correlated electron system, the electron–electron interaction induces a short range magnetic ordering which, in turn, introduces a molecular field on the magnetic impurity [16]. Therefore, the impurity and a electron form a triplet aligned to the molecular field, thus, resulting the broken local SU(2) symmetry. These equations are naturally reduced to the scaling equations of the conventional Kondo model when there is no Luttinger interaction i.e. $\eta_\sigma = 1$ and $\eta_\rho = 1$.

So far, no assumption has been made on the strength of $\eta_\rho (= [(1 - g/\pi v_F)/(1 + g/\pi v_F)]^{1/2})$. Here, g is the strength of scattering between the right-going and the left-going mode and the scattering within the same mode is neglected. In the small g regime, the above scaling equations are reduced as follows:

$$\frac{\mathrm{d}(\rho_0 J_{zF})}{\mathrm{d}l} = (\rho_0 J_{\perp F})^2 + (\rho_0 J_{\perp B})^2 - \frac{g}{2\pi v_F}(\rho_0 J_{\perp B})^2$$
$$\frac{\mathrm{d}(\rho_0 J_{zB})}{\mathrm{d}l} = \frac{g}{2\pi v_F}(\rho_0 J_{zB}) + 2(\rho_0 J_{\perp F})(\rho_0 J_{\perp B})$$
$$\frac{\mathrm{d}(\rho_0 J_{\perp F})}{\mathrm{d}l} = (\rho_0 J_{zF})(\rho_0 J_{\perp F}) + (\rho_0 J_{zB})(\rho_0 J_{\perp B}) - \frac{2g}{2\pi v_F}(\rho_0 J_{zB})(\rho_0 J_{\perp B})$$
$$\frac{\mathrm{d}(\rho_0 J_{\perp B})}{\mathrm{d}l} = \frac{g}{2\pi v_F}(\rho_0 J_{\perp B}) + (\rho_0 J_{zF})(\rho_0 J_{\perp B}) + (\rho_0 J_{zB})(\rho_0 J_{\perp F}). \quad (36)$$

We note that the above result is in agreement with the FN poor man's scaling result except for the symmetry breaking terms involving the g parameter. In fact, the extra terms in our RG equations correspond to the next higher-order terms which were neglected in the FN approach.

4. The ground state

The scaling equations yield two strong coupling fixed points, $(J_F, J_B) = (\infty, \infty)$ and $(\infty, -\infty)$. The first fixed point governs the antiferromagnetic regime, which gives the singlet as the ground state. The second one corresponds to the ferromagnetic coupling. However it is not clear whether this fixed point corresponds to a singlet or to a triplet state. We have calculated flows of the coupling constant for several values of the Luttinger interaction parameter, η, for the ferromagnetic fixed point. For finite η other than unity, $J_B/J_F < -1$ and $J_B + J_F$ flows to $-\infty$, whereas $J_B + J_F$ is equal to zero when $\eta = 1$ (figure 1). When the coupling constant J grows and becomes large, the Kondo coupling term becomes dominant in equation (35). Therefore, the Luttinger interaction becomes irrelevant [7] and we can treat the impurity spin as a classical spin with the magnitude $S = 1/2$.

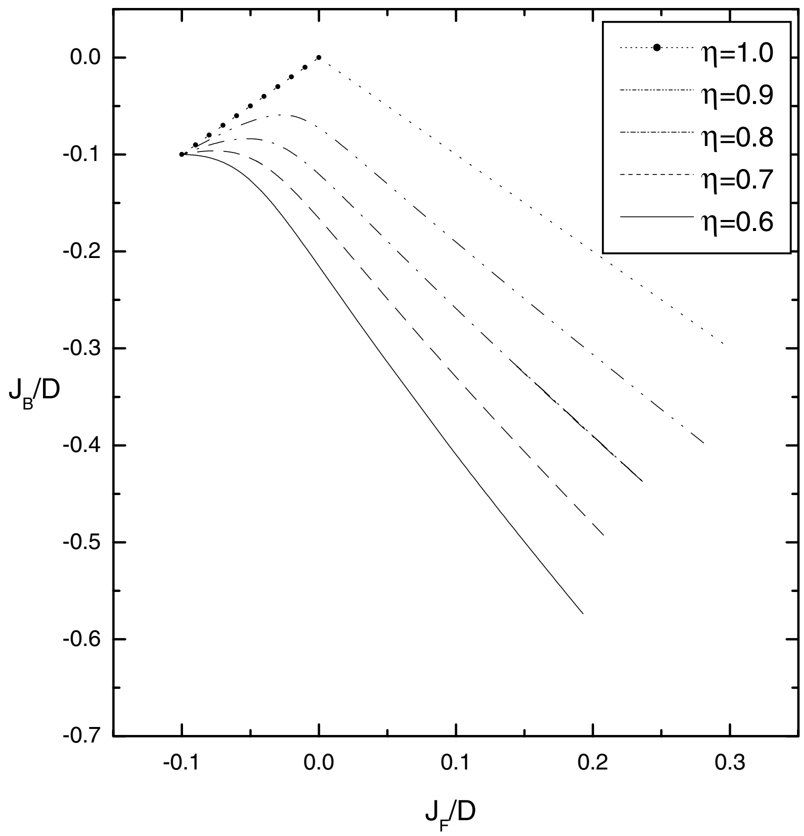

Figure 1. The coupling constant $J_{F,B}$ for several Luttinger interaction strengths η. The magnitude of slope and the value of $|J_F + J_B|$ become larger as the interaction strength grows larger. The slope of the dotted line is -1.

The asymptotic Hamiltonian which represents this situation can be written as

$$H = \sum_{rk\sigma} \epsilon_k c^{\dagger}_{rk\sigma} c_{rk\sigma} + \frac{1}{4} \sum_{rk,r'k',\sigma} J_{rr'\sigma} c^{\dagger}_{rk\sigma} c_{r'k'\sigma'} \tag{37}$$

where $J_{11\sigma} = J_{22\sigma} = \sigma J_F$ and $J_{12\sigma} = J_{21\sigma} = \sigma J_B$. The Luttinger interaction effect is included in the coupling constant through the renormalization process. The corresponding Green functions are given as follows:

$$G_{11}(\epsilon) = G_{22}(\epsilon) = G_0(\epsilon) + G_0(\epsilon) \frac{\frac{1}{4}\sigma J_F}{1 - \frac{1}{4}\sigma(J_F + J_B)G_0(\epsilon)} G_0(\epsilon)$$

$$G_{12}(\epsilon) = G_{21}(\epsilon) = G_0(\epsilon) \frac{\frac{1}{4}\sigma J_B}{1 - \frac{1}{4}\sigma(J_F + J_B)G_0(\epsilon)} G_0(\epsilon). \tag{38}$$

For $\eta = 1 (J_F + J_B = 0)$, there exists no bound state since the Green functions do not have poles. This case has the same properties as the three-dimensional ferromagnetic Kondo coupling case. However, if the Luttinger interaction is turned on ($\eta < 1$), $J_F + J_B$ flows to $-\infty$. The Green functions have a pole for $\sigma = +1$ suggesting that a conduction electron becomes bounded [17] to form a triplet state in the ground state for the ferromagnetic Kondo exchange.

5. The scalar potential scattering effect

In a more realistic model, a magnetic impurity generates scattering due to an elastic potential, ω [10]. Recently, Wang and his coworkers have carried out Bethe *ansatz* calculations on the one-dimensional Kondo problem [8, 9]. They showed that for an attractive potential scattering, the bound state always appears, whereas for a repulsive potential scattering, there exists a bound state for $|J| > 4\omega$, but no bound state for $|J| < 4\omega$.

The scaling theory for the scalar potential in the first order is similar to the magnetic interaction. The bosonized term is

$$\frac{1}{2\pi\alpha}(2\omega_F\alpha\partial_x\sqrt{2}\phi_\rho(0) + 4\mathrm{i}\omega_B\eta_{+\uparrow}\eta_{-\uparrow}\cos(\sqrt{2}\phi_\rho(0))\sin(\sqrt{2}\phi_\sigma(0))) \tag{39}$$

and we have the scaling equations

$$\frac{\mathrm{d}\omega_F}{\mathrm{d}l} = 0 \qquad \frac{\mathrm{d}\omega_B}{\mathrm{d}l} = \frac{1-\eta_\rho}{2}\omega_B \tag{40}$$

showing that the backscattering contribution is relevant. In the second order, the candidates for renormalizing the forward potential scattering are contributions from the terms like ω_F^2 or ω_B^2. But, we find that ω_F^2 yields only a constant by a simple calculation,

$$\frac{1}{(2\pi\alpha)^2}4\omega_F^2\alpha^2\int \mathrm{d}\tau\int \mathrm{d}\tau'\,\langle\partial_x\sqrt{2}\phi_{\rho_f}(\tau)\partial_x\sqrt{2}\phi_{\rho_f}(\tau')\rangle \tag{41}$$

which clearly has no slow mode. Similarly, the ω_B^2 term generates terms such as $\cos(2\sqrt{2}\phi_\sigma(\tau))$, and $\cos(2\sqrt{2}\phi_\rho(\tau))$, which are irrelevant as discussed above. Also, we can show that J_{zF}^2, J_{zB}^2, $J_{zF}J_{zB}$, w_Fw_B and $J_{z\perp,FB}\omega_{FB}$ are also irrelevant through the same procedure as in ω_F^2 and ω_B^2. Thus, the potential scattering is not scaled at the second-order cumulant and there is no interplay between the magnetic and the non-magnetic interaction. Actually, this situation is not unexpected, since we know that the nearest neighbour Coulomb interaction in the Hubbard model does not affect the magnetic properties [18].

In the case of an impurity spin, $S = 1/2$, the energy of a single electron coupled with the impurity is such that the ferromagnetic coupling energy is $\frac{1}{4}J$ and the scalar potential is ω. It means that if $4\omega > |J|$ initially, the potential energy grows infinitely along the scaling process so that no bound state can be formed. If $4\omega < |J|$ initially, a spin triplet state is formed. Under the open boundary condition, this corresponds to a bound state of a spin $\frac{3}{2}$ complex in agreement with the Bethe *ansatz* results [8]. Here, we note that, in the present treatment, it is not necessary to invoke the nearest neighbour spin interaction as done by FN. We believe that the short-ranged nature of the spin interaction makes the FN scenario unlikely, although it cannot be ruled out completely.

6. The Kondo temperature

In the conventional three-dimensional Kondo model, the Kondo temperature is given by $T_K = D\,\mathrm{e}^{-1/2J}$, which originates from the scaling equation $\mathrm{d}J/\mathrm{d}\ln D = -2J^2$, where D is the bandwidth. In their previous study, Lee and Toner showed that the scaling relation, $\mathrm{d}J_B/\mathrm{d}\ln D = -((1-\eta)/2)J_B$, gives a power law Kondo temperature, $T_K = DJ_B^{2/(1-\eta)}$ [6]. However, the crossover behaviour from the Fermi liquid to the Luttinger one has not been studied. In order to address this question, we consider the scaling to second order for the forward and the backward scattering simultaneously.

From the RG flow, we get the coupling constants as functions of D, i.e. $\tilde{J}_F(\tilde{D})$ and $\tilde{J}_B(\tilde{D})$, where $\tilde{J}_F$ and $\tilde{J}_B$ are the scaled couplings and $\tilde{D}$ is the scaled bandwidth cut-off. The Kondo

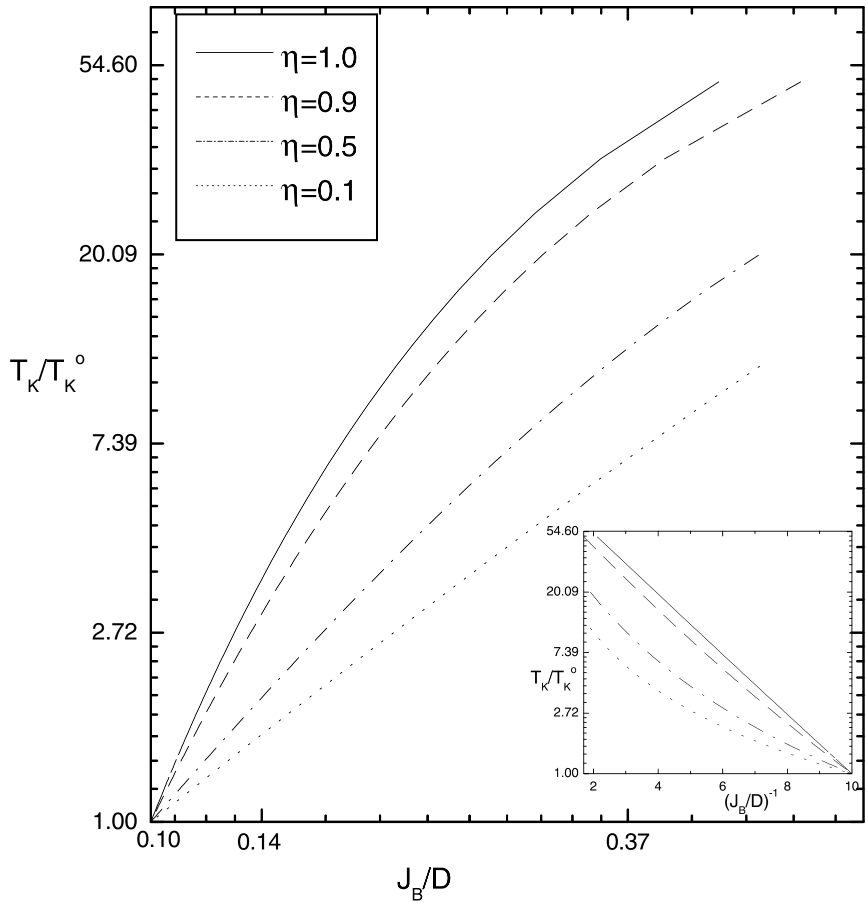

Figure 2. The Kondo temperature as a function of the backward scattering coupling constant, J_B, for several Luttinger interaction strengths η, where T_K^o is the Kondo temperature for each η at $J_B/D = 0.1$. The linear to nonlinear crossover is clearly shown. Note the log–log scale of the graph. The inset shows the Kondo temperature as a function of J_B^{-1}. The linear curve for $\eta = 1$ clearly shows the $T_K = D\mathrm{e}^{-1/2J}$ behaviour.

temperature is an invariant energy scale in the scaling procedure and, thus, can be expressed as $T_K = \tilde{D} f(\tilde{J})$, where $f(\tilde{J})$ becomes exponential or power-law type depending on the limiting case. For some initial value of J, T_K remains constant through the scaling procedure. In such a case, $f(\tilde{J})$ becomes proportional to $1/\tilde{D}(\tilde{J})$, thus yielding the Kondo temperature.

T_K for the backward scattering part is given in figure 2. It can be clearly seen that T_K is exponential for a weak Luttinger interaction and becomes power-law type as the Luttinger interaction increases. From inset, we observe that the linear slope of the linear plot ($\eta = 1$) is $-1/2$ in agreement with $T_K = D\,\mathrm{e}^{-1/2J}$.

7. Summary

In summary, we have studied the Kondo effect in a Luttinger liquid in the presence of a scalar potential. We have obtained the scaling equations up to the second order for an arbitrary Luttinger interaction strength by renormalizing the boson fields.

The ferromagnetic fixed point is studied using an asymptotic Hamiltonian. It is shown that a triplet bound state can be formed in agreement with the recent Bethe *ansatz* calculation without invoking the nearest neighbour spin interaction. The Luttinger interaction induce the triplet state to break the local SU(2) symmetry about the impurity spin contrary to the result obtained by Furusaki and Nagaosa. The magnetic interaction and the potential scattering do not interplay and the triplet state is sustained for a weak scalar potential, $|J| > 4\omega$. The Kondo temperature for arbitrary strength of the Luttinger interaction is calculated. The result shows a clear crossover behaviour from an exponential to a power law type.

Acknowledgments

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